Bobby Bonilla's Contract Explained

In 2000, Bobby Bonilla was released by the New York Mets, who owed him $5.9 million. The Mets agreed to pay it out in 25 installments between 2011 and 2035 inclusive. The original $5.9 million would grow at 8% compounded annually for 11 years (2000-2010 inclusive).

The table below was found online, an addendum to his contract that shows exactly how the money accrues interest then is amortizes to 0. This sheet was found online, a part of a copy of his contract being auctioned at www.goldin.co.

Bonilla

EXHIBIT A

Payout over 25 yrs. Beginning 07/01/11

Bonilla earns 8% on money from 2000 through depletion. Based on annual compounding.

PayOut	Starting Balance	Annual Amort.	Bal. After Pay	Interest Earned
7/1/2011	13,756,670	(1,193,248)	12,563,422	1,005,074
7/1/2012	13,568,496	(1,193,248)	12,375,247	990,020
7/1/2013	13,365,267	(1,193,248)	12,172,019	973,762
7/1/2014	13,145,781	(1,193,248)	11,952,532	956,203
7/1/2015	12,908,735	(1,193,248)	11,715,487	937,239
7/1/2016	12,652,726	(1,193,248)	11,459,477	916,758
7/1/2017	12,376,236	(1,193,248)	11,182,987	894,639
7/1/2018	12,077,626	(1,193,248)	10,884,378	870,750
7/1/2019	11,755,129	(1,193,248)	10,561,880	844,950
7/1/2020	11,406,831	(1,193,248)	10,213,583	817,087
7/1/2021	11,030,669	(1,193,248)	9,837,421	786,994
7/1/2022	10,624,415	(1,193,248)	9,431,166	754,493
7/1/2023	10,185,660	(1,193,248)	8,992,412	719,393
7/1/2024	9,711,804	(1,193,248)	8,518,556	681,485
7/1/2025	9,200,041	(1,193,248)	8,006,793	640,543
7/1/2026	8,647,336	(1,193,248)	7,454,088	596,327
7/1/2027	8,050,415	(1,193,248)	6,857,167	548,573
7/1/2028	7,405,740	(1,193,248)	6,212,492	496,999
7/1/2029	6,709,491	(1,193,248)	5,516,243	441,299
7/1/2030	5,957,542	(1,193,248)	4,764,294	381,144
7/1/2031	5,145,438	(1,193,248)	3,952,189	316,175
7/1/2032	4,268,365	(1,193,248)	3,075,116	246,009
7/1/2033	3,321,126	(1,193,248)	2,127,878	170,230
7/1/2034	2,298,108	(1,193,248)	1,104,860	88,389
7/1/2035	1,193,248	(1,193,248)	0	0

At 8% Bonilla payment = $1,193,248 for 25 years beginning July 1, 2011.

Bonilla Fund's grow until initial payment as below. 8.00%

		Interest Earned	End Balance
2000	5,900,000	472,000	6,372,000
2001	6,372,000	509,760	6,881,760
2002	6,881,760	550,541	7,432,301
2003	7,432,301	594,584	8,026,885
2004	8,026,885	642,151	8,669,036
2005	8,669,036	693,523	9,362,559
2006	9,362,559	749,005	10,111,563
2007	10,111,563	808,925	10,920,488
2008	10,920,488	873,639	11,794,127
2009	11,794,127	943,530	12,737,657
2010	12,737,657	1,019,013	13,756,670

The bottom half of the sheet shows the accrual period. The principal P is the \$5.9 million, the annual percentage rate r is 8%, written 0.08, and the period t is in years. Note that 2000-2010 inclusive is 11 years. The formula for the future value of compounded interest on a principal is

$$PV = P(1+r)^t.$$

Using the formula, substitute in the values for P, r and t:

$$PV = P(1+r)^t = 5{,}900{,}000(1.08)^{11} = \$13{,}756{,}670.$$

This is the figure shown on the bottom right of the sheet. This is called the “Present Value” (abbreviated PV) because as of 2011, when the payments to Bonilla begin, it will be considered the present value of the fund used to pay him.

Let’s assume for the moment we don’t know how much Bonilla is to be paid every year starting in 2011. Call this lower-case p.

The Present Value, \$13,756,670, will be amortized over a period of 25 years until the value of the fund is \$0. It's important to state that the Present Value is not static. It will not be divided into 25 equal chunks to be paid to Bonilla. In amortization, the Present Value continues to grow, while at regular intervals, a payment (p) is subtracted. So it is a consistent "growth-subtraction" routine that plays out over a 25-year term, where the last subtraction brings the fund to 0.

In the first year (2011, denoted $n = 1$ so that we can track the number of years easier), the first payment p is subtracted from the present value PV. This is simply the expression $PV - p$. This is the new value of the Bonilla fund heading into 2012.

In the second year (2012, or $n = 2$), two things happen in the following order: (1) it accrues interest by 8%, and (2) then payment p is subtracted. This is the new value of the Bonilla fund heading into 2013. This is written as the expression

$$1.08(PV - p) - p.$$

This can be described in more detail. To “increase by 8%” means to multiply the fund’s value ($PV - p$) by 1.08, so we have $1.08(PV - p)$; then from that, payment p is subtracted. That’s how we get $1.08(PV - p) - p$. This can be simplified by multiplying to clear parentheses: $1.08PV - 1.08p - p$.

In the third year (2013, $n = 3$), the same two things happen: (1) the fund’s value from 2012 accrues 8% interest, and (2) payment p is subtracted. The value of the Bonilla fund heading into 2014 is

$$1.08\,\underbrace{(1.08PV - 1.08p - p)}_{\substack{\text{This is the expression from} \\ \text{the previous year.}}} - p.$$

Multiplying to clear parentheses, this is $1.08^2PV - 1.08^2p - 1.08p - p$.

In the fourth year (2014, $n = 4$), we continue the same scheme. Note that the expression from one year gets "fitted" into the expression. Thus, the value of the Bonilla fund heading into 2015 is

$$1.08(1.08^2 PV - 1.08^2 p - 1.08p - p) - p.$$

Multiplying to clear parentheses, this is

$$1.08^3 PV - 1.08^3 p - 1.08^2 p - 1.08p - p.$$

Let's collect this into a table and note a pattern:

Year	***n*-value**	**Value of the fund after payment**
2011	1	$PV - p$
2012	2	$1.08PV - 1.08p - p$
2013	3	$1.08^2 PV - 1.08^2 p - 1.08p - p$
2014	4	$1.08^3 PV - 1.08^3 p - 1.08^2 p - 1.08p - p$

The pattern suggests a way to describe the value of the Bonilla fund without having to go through the intermediate steps. For example, in 2015 ($n = 5$), the value of the fund will be

$$1.08^4 PV - 1.08^4 p - 1.08^3 p - 1.08^2 p - 1.08p - p.$$

Observe that the highest exponent of the terms is always one less than the n-value representing the given year. Thus, in 2035 ($n = 25$), the value of the Bonilla fund would be written

$$1.08^{24} PV - 1.08^{24} p - 1.08^{23} p - 1.08^{22} p - \cdots - 1.08^3 p - 1.08^{2p} - 1.08p - p.$$

Factor $-p$ from the rightmost terms. This gives:

$$1.08^{24} PV - p(1.08^{24} + 1.08^{23} + 1.08^{22} + \cdots + 1.08^3 + 1.08^2 + 1.08 + 1).$$

The terms within the parentheses are a geometric series. Reading right to left, the first term is $a = 1$, and the common ratio is $c = 1.08$. The formula for the sum of the first n terms of a geometric series is

$$S = \frac{a(c^n - 1)}{c - 1}.$$

Note that the series $1.08^{24} + 1.08^{23} + 1.08^{22} + \cdots + 1.08^3 + 1.08^2 + 1.08 + 1$ has 25 terms. Thus, the terms within the parentheses can be summaries as a geometric sum:

$$S = \frac{1(1.08^{25} - 1)}{1.08 - 1}, \quad \text{which simplifes to} \quad S = \frac{1.08^{25} - 1}{0.08}.$$

Finally, the long expression representing the value of the Bonilla fund in 2035 is

$$1.08^{24} PV - p\left(\frac{1.08^{25} - 1}{0.08}\right).$$

The plan was that the value after the last payment would be \$0, and the fund fully amortized. This expression above is set to 0 and solved for p:

$$1.08^{24} PV - p\left(\frac{1.08^{25} - 1}{0.08}\right) = 0 \qquad \text{so that} \qquad p = \frac{1.08^{24} PV}{\left(\frac{1.08^{25} - 1}{0.08}\right)}.$$

Substituting $PV = \$13{,}756{,}670$ and using a calculator, we get $p = \$1{,}193{,}248.19$. And this is what Mr. Bonilla receives every year between 2011 and 2035. This is shown in the table on the first page.