MAT 267 – Exam 1 – Surgent – February 13, 2025 Name: Key

Show all work and be neat. You may use the backside for scratch, but work written on the backside won't be looked at. Place all work you want graded on the front. You agree to abide by all aspects of the honor code while taking this exam and will not discuss the contents of this exam during the examination period.

PART I: Multiple Choice. Write the letter of your choice in the space provided at right. (6 pts each)

1. The sphere $(x-4)^2+(y+5)^2+(z+1)^2=41$ intersects the yz plane and forms a circle. What is the radius of this circle

Ans(1) D

A. 25 **B.** 41 **C.** $\sqrt{41}$ **D.** 5

Set $x=0$, you get $(-4)^2=16$, subtracted from 41 gives 25, its square root is 5.
The other form, set $y=0$, the radius is 4.

2. Given $\mathbf{u}=\langle 3,-1,4\rangle$. The vector parallel to $\mathbf{u}$ and of length 5 is

Ans(2) A

A. $\langle \frac{15}{\sqrt{26}},-\frac{5}{\sqrt{26}},\frac{20}{\sqrt{26}}\rangle$ **B.** $\langle \frac{15}{\sqrt{24}},-\frac{5}{\sqrt{24}},\frac{20}{\sqrt{24}}\rangle$ **C.** $\langle 15,-5,20\rangle$ **D.** $\langle -15,5,-20\rangle$

Find the unit vector then multiply by 5

3. At what angle do the planes $4x+7y-z=12$ and $5x+3y-2z=8$ intersect?

Ans(3) D

A. $36.33°$ **B.** $32.05°$ **C.** $38.85°$ **D.** $30.84°$

Find the two normal and find the angle between them

4. Let $\mathbf{u}=\langle 6,k,1\rangle$ and $\mathbf{v}=\langle 4,5,k\rangle$. For what interval of k are $\mathbf{u}$ and $\mathbf{v}$ obtuse?

Ans(4) A

A. $(-\infty,-4)$ **B.** $(-4,\infty)$ **C.** $(4,\infty)$ **D.** $(-\infty,4)$

Dot: $24+6k$. For obtuse, you want $24+6k<0$ so that $k<-4$. For acute, $k>-4$.

5. Find the equation of the line passing through $A=(4,2,-1)$ and $B=(-3,1,5)$ such that $t=0$ gives A and $t=1$ gives B.

Ans(5) C

A. $\langle 4-3t,2+t,-1+5t\rangle, 0\le t\le 1$ **B.** $\langle -3+4t,1+2t,5-t\rangle, 0\le t\le 1$
C. $\langle 4-7t,2-t,-1+6t\rangle, 0\le t\le 1$ **D.** $\langle -3+7t,1-t,5-6t\rangle, 0\le t\le 1$

6. Let $\mathbf{r}(t)=\langle 3t^2+t,2t-3,t^3\rangle$ trace a curve in R^3. Find $\mathbf{T}(2)$.

Ans(6) C

A. $\langle 13,2,12\rangle$ **B.** $\langle 6,0,12\rangle$ **C.** $\frac{1}{\sqrt{317}}\langle 13,2,12\rangle$ **D.** $\frac{1}{\sqrt{180}}\langle 6,0,12\rangle$

Find $r'(2)$ then divide by the magnitude.

7. Two non-zero vectors **u** and **v** are parallel. What is true about $\mathbf{u} \times \mathbf{v}$?

Ans(7) A

A. It is $\mathbf{0}$ **B.** It is the product of their lengths
C. It points out of the page **D.** Not enough information

For the dot product, the answer is B.

8. Where does the line $\langle 2-t, 1+3t, 3+4t\rangle$ intersect the plane $3x+5y-z=24$?

Ans(8) C

A. $t=2$ **B.** $t=-2$ **C.** $(0,7,11)$ **D.** $(4,-5,-5)$

Plug in the components and solve for t, which is 2, then plug the 2 in to get the point.
For the other form, $t=-2$ and the point was D.

9. The position of an object is given by $\mathbf{r}(t) = \langle t^2, 5t, t^3\rangle$. Find the object's acceleration at $t=4$ sec.

Ans(9) C

A. $\langle 16,20,64\rangle$ **B.** $\langle 8,5,48\rangle$ **C.** $\langle 2,0,24\rangle$ **D.** 48.92

Acceleration is the second derivative of r. Then evaluate at $t=4$.

Part II: Free response. Show all work and be neat! Any work you want looked at for grading purposes must be written within the space provided for the problem. Place your answer in the space provided.

10. Given three points $P=(5,4,3)$, $Q=(2,4,-3)$ and $R=(7,0,-5)$ in R^3. Find the equation of the plane passing through these points. Leave your answer in $Ax+By+Cz=D$ form.

Vectors: $PQ = \langle -3,0,-6\rangle$ & $PR = \langle 2,-4,-8\rangle$. The cross product is $\langle -24,-36,12\rangle$. CHECK IT!!!!

The initial form is $-24(x-5)-36(y-4)+12(z-3)=0$.

When simplified, the equation is $-24x-36y+12z=-228$, or reduced, $2x+3y-z=19$.

11. Let $\mathbf{r}(t) = \langle 4t, 5\cos(t), 5\sin(t)\rangle$ trace a curve in R^3.

a) Find the equation (in vector form) of the tangent line to **r** at $t=\frac{\pi}{2}$. All components of the answer must be simplified (no trigonometric forms left over).

$\mathbf{r}\left(\frac{\pi}{2}\right) = \langle 4\left(\frac{\pi}{2}\right), 5\cos\left(\frac{\pi}{2}\right), 5\sin\left(\frac{\pi}{2}\right)\rangle = \langle 2\pi, 0, 5\rangle$.
$\mathbf{r}'(t) = \langle 4, -5\sin(t), 5\cos(t)\rangle$, so that $\mathbf{r}'\left(\frac{\pi}{2}\right) = \langle 4, -5\sin\left(\frac{\pi}{2}\right), 5\cos\left(\frac{\pi}{2}\right)\rangle = \langle 4,-5,0\rangle$.
The line is $\langle 2\pi, 0, 5\rangle + t\langle 4,-5,0\rangle$, or $\langle 2\pi+4, -5t, 5\rangle$

b) Find the exact length of the arc traced between $t=0$ and $t=\frac{\pi}{2}$.

$|\mathbf{r}'(t)| = \sqrt{(4)^2+(-5\sin(t))^2+(5\cos(t))^2} = \sqrt{41}$, so that length $= \int_0^{\pi/2} \sqrt{41}\,dt = \sqrt{41}\left(\frac{\pi}{2}\right)$.

12. Starting at the origin, a helicopter is flying toward the point (4,7) on a straight line. The actual landing pad is at (8,2). The helicopter is allowed one right-angle turn to arrive at the landing pad. Find the point at which the helicopter makes this turn.

The vectors are $\mathbf{u} = \langle 4,7 \rangle$ and $\mathbf{v} = \langle 8,2 \rangle$. Vector $\mathbf{v}$ is the hypotenuse, so that $\mathbf{u}$ is the adjacent leg. Project $\mathbf{v}$ onto $\mathbf{u}$:

$$\left(\frac{\langle 8,2 \rangle \cdot \langle 4,7 \rangle}{\langle 4,7 \rangle \cdot \langle 4,7 \rangle}\right)\langle 4,7 \rangle = \left(\frac{46}{65}\right)\langle 4,7 \rangle = \left(\frac{184}{65}, \frac{322}{65}\right) \approx (2.831, 4.954).$$

The other form, the vectors were swapped, you wanted to project $\mathbf{u}$ onto $\mathbf{v}$.

$$\left(\frac{\langle 8,2 \rangle \cdot \langle 4,7 \rangle}{\langle 8,2 \rangle \cdot \langle 8,2 \rangle}\right)\langle 8,2 \rangle = \left(\frac{46}{68}\right)\langle 8,2 \rangle = \left(\frac{184}{34}, \frac{46}{34}\right) \approx (5.412, 1.278).$$

13. A particle's velocity is given by $\mathbf{v}(t) = \langle 2t, e^{3t}, \cos(2t) \rangle$. Find its position vector function where the initial condition is $\mathbf{r}(0) = \langle 4,1,5 \rangle$.

Integrate:

$$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int \langle 2t, e^{3t}, \cos(2t) \rangle\,dt = \langle t^2 + A, \tfrac{1}{3}e^{3t} + B, \tfrac{1}{2}\sin(2t) + C \rangle.$$

At $t = 0$, you have $\langle 0^2 + A, \frac{1}{3}e^{3(0)} + B, \frac{1}{2}\sin(2(0)) + C \rangle = \langle A, \frac{1}{3} + B, C \rangle = \langle 4,1,5 \rangle$ so that $A = 4, B = \frac{2}{3}, C = 5$.

The position vector is

$$\mathbf{r}(t) = \langle t^2 + 4, \tfrac{1}{3}e^{3t} + \tfrac{2}{3}, \tfrac{1}{2}\sin(2t) + 5 \rangle.$$

The other form had $\mathbf{v}(t) = \langle 3t, e^{2t}, \cos(4t) \rangle$, same initial condition, and the position vector is

$$\mathbf{r}(t) = \langle \tfrac{3}{2}t^2 + 4, \tfrac{1}{2}e^{2t} + \tfrac{1}{2}, \tfrac{1}{4}\sin(4t) + 5 \rangle.$$