

Quiz 2 key

Part A

Question 1

1 / 1

Let $u = \langle k, 5, -2 \rangle$ and $\langle 1, k, 9 \rangle$ be two vectors in R^3 . Which statement below is true?

☒ u and v are acute when $k > 3$, obtuse when $k < 3$ and orthogonal when $k = 3$.

☐ u and v are obtuse when $k > -3$, acute when $k < -3$ and orthogonal when $k = -3$

☐ u and v are acute when $k > -3$, obtuse when $k < -3$ and orthogonal when $k = -3$

☐ u and v are obtuse when $k > 3$, acute when $k < 3$ and orthogonal when $k = 3$.

Dot product: $\langle k, 5, -2 \rangle \cdot \langle 1, k, 9 \rangle = 6k - 18$. Set to 0 gives $k = 3$. Thus, orthogonal when $k = 3$, acute when $k > 3$ and obtuse when $k < 3$.

Question 2

1

Find the angle between vectors $u = \langle 2, 7 \rangle$ and $v = \langle -1, 8 \rangle$. Give the angle in degrees to two decimal places.

☐ 20.85 degrees

☒ 23.07 degrees

☐ 0.40 degrees

☐ 8.82 degrees

$$\theta = \cos^{-1} \left(\frac{\langle 2, 7 \rangle \cdot \langle -1, 8 \rangle}{|u||v|} \right) = \cos^{-1} \left(\frac{52}{\sqrt{53}\sqrt{65}} \right) \approx 23.07^\circ$$

Question 3

1

Vector $u = \langle 6, 5 \rangle$ is projected onto $v = \langle 9, 2 \rangle$. Find the projection vector.

☐ $(64/61)\langle 9, 2 \rangle$
☒ $(64/85)\langle 9, 2 \rangle$
☐ $(64/85)\langle 6, 5 \rangle$
☐ $(64/61)\langle 6, 5 \rangle$

$$\text{Projection} = \frac{u \cdot v}{v \cdot v} v = \frac{64}{85} v.$$

Question 4

1

Let $u = \langle 2, -1, 4 \rangle$ and $v = \langle -3, 2, 1 \rangle$ be two vectors in $\mathbb{R}^3$. Find $u \times v$.

☒ $-9i - 14j + k$
☐ $9i - 14j + k$
☐ $-9i + 14j + k$
☐ $9i + 14j + k$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & -1 & 4 \\ -3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} k = -9i - 14j + k. \text{ CHECK IT!}$$

Question 5

1

/ 1 pt

Same two vectors as in Question 4. Find the area of the triangle formed by these two vectors.

☐ $\sqrt{278}$
☒ $(1/2)\sqrt{278}$
☐ $\sqrt{276}$
☐ $(1/2)\sqrt{276}$

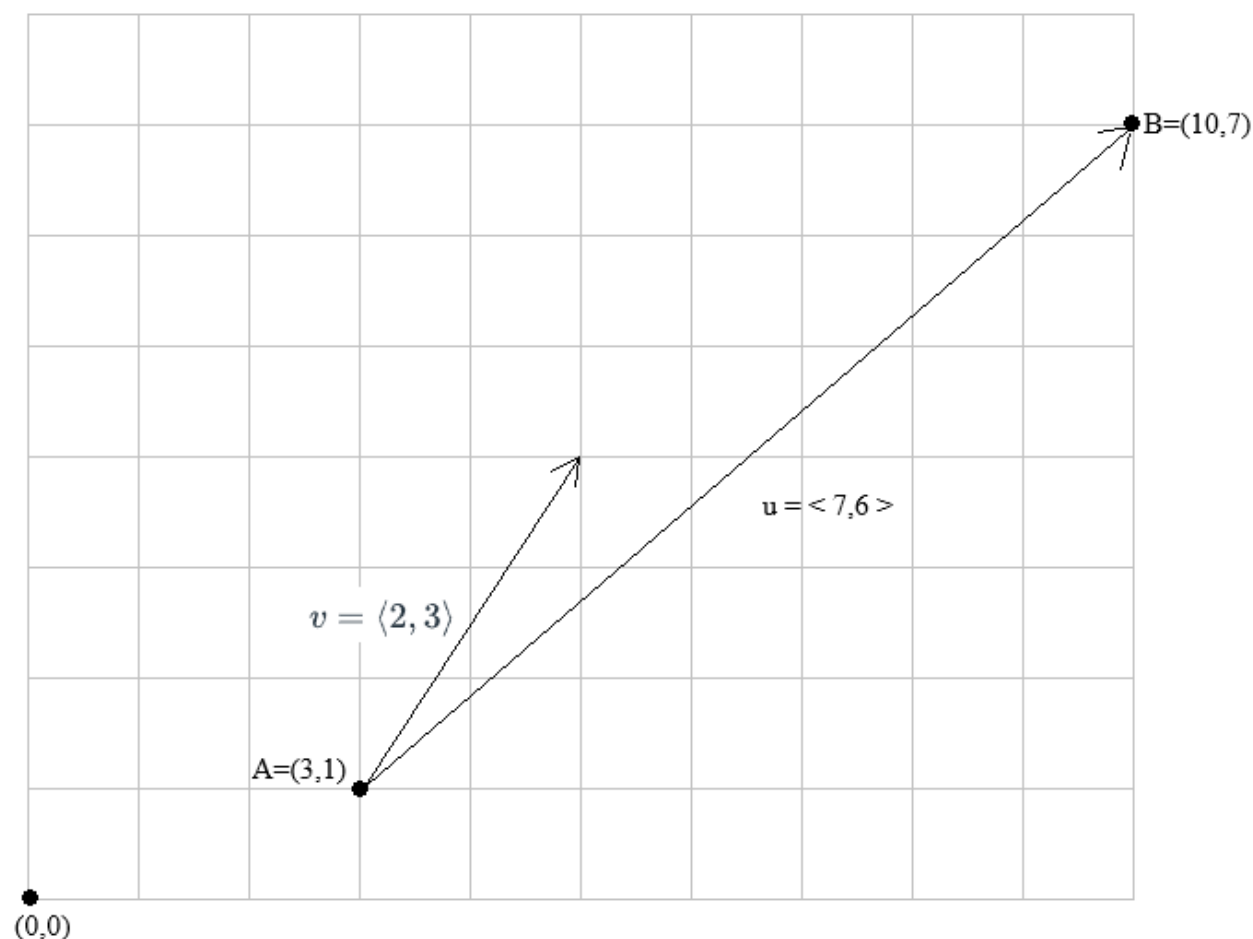
$$\text{Triangle area} = \frac{1}{2} \sqrt{(-9)^2 + (-14)^2 + (1)^2} = \frac{1}{2} \sqrt{278}.$$

Part B:

A hiker starts hiking at point $A = (3,1)$, heading in the direction of vector $v = \langle 2, 3 \rangle$. The hiker's finish point is $B = (10,7)$. He is allowed one right-angle turn.

- (a) [3 pts] At what point does the hiker make this right-angle turn so as to finish at B?
- (b) [1 pt] How far did he walk in the direction of v ?
- (c) [1 pt] How far did he walk in the direction orthogonal to v ? (i.e. after he made the turn)

Give all answers to two decimal places.



Create vector u from A to B . Vector u is the hypotenuse, project u onto v : $\frac{u \cdot v}{v \cdot v} v = \frac{32}{13} \langle 2, 3 \rangle = \langle \frac{64}{13}, \frac{96}{13} \rangle \approx \langle 4.92, 7.38 \rangle$.

- (a) The turn occurs at $(3+4.92, 1+7.38) = (7.92, 8.38)$.
- (b) Distance in direction of v is the magnitude of the projection, $\sqrt{4.92^2 + 7.38^2} \approx 8.87$ units.
- (c) The norm (or opposite) leg is $u - \text{projection} = \langle 7, 6 \rangle - \langle 4.92, 7.38 \rangle = \langle 2.08, -1.38 \rangle$. Thus, the distance is $\sqrt{2.08^2 + (-1.38)^2} \approx 2.5$ units.