

Quiz 3 key

Part A

Question 1

Parameterize the line segment from $A = (-1, 5, 2)$ to $B = (4, -6, 1)$.

☒ $\langle -1+5t, 5-11t, 2-t \rangle, t: [0,1]$

☐ $\langle -1+5t, 5-11t, 2-t \rangle$

☐ $\langle 4-5t, -6+11t, 1+t \rangle, t: [0,1]$

☐ $\langle 4-5t, -6+11t, 1+t \rangle$

Vector $AB = \langle 4 - (-1), -6 - 5, 1 - 2 \rangle = \langle 5, -11, -1 \rangle$, using A, line is $L: \langle -1, 5, 2 \rangle + t \langle 5, -11, -1 \rangle$, t from 0 to 1. If it is a segment, include bounds. If not, bounds are not necessary.

Correct answer

Question 2

Consider the two lines in R^3 :

$$L_1 : \langle 2 - t, 5 + 2t, 3 + 4t \rangle$$

$$L_2 : \langle 3 - s, -1 + 3s, -5 + 5s \rangle$$

Find the intersection point of these two lines.

☒ $(-1, 11, 15)$

☐ $(1, -11, -15)$

☐ $(2, 5, 3)$

☐ They are parallel

Set the x and y parts equal: $\begin{aligned} 2 - t &= 3 - s & \rightarrow & s - t = 1 \\ 5 + 2t &= -1 + 3s & \rightarrow & -3s + 2t = -6 \end{aligned} \rightarrow \begin{aligned} s &= 4 \\ t &= 3 \end{aligned}$ (any solution method is fine). Evaluate the lines at s or t, get $(-1, 11, 15)$

Correct answer

Question 3

1 /

Let $A = (2,1,3)$, $B = (4,-2,1)$ and $C = (8,0,3)$ be three points in $\mathbb{R}^3$. Find the equation of the plane that is formed by these three points.

☒ $x + 6y - 8z = -16$

☐ $x + 6y - 8z = 16$

☐ $x - 6y - 8z = -16$

☐ $x + 6y + 8z = -16$

Get 2 vectors, $AB = \langle 2, -3, -2 \rangle$ and $AC = \langle 6, -1, 0 \rangle$. Their cross product is $n = \langle -2, -12, 16 \rangle$ CHECK IT!!!. Using point A, the plane is $-2(x - 2) - 12(y - 1) + 16(z - 3) = 0$. After simplification, $x + 6y - 8z = -16$.

Correct answer

Question 4

1

Where does the line $L : \langle 1 + 2t, 3 - t, 5 + 4t \rangle$ intersect the plane $3x - 4y + 5z = 76$?

☒ $(5,1,13)$

☐ $t = 2$

☐ $\langle 3, -4, 5 \rangle$

☐ $(5,-1,-13)$

Substitute: $3(1 + 2t) - 4(3 - t) + 5(5 + 4t) = 76$. Simplify: $3 + 6t - 12 + 4t + 25 + 20t = 76$. Simplify some more: $6t + 4t + 20t = 76 - 16 \rightarrow 30t = 60 \rightarrow t = 2$. Plug back into L, get $(5,1,13)$

Correct answer

Question 5

1

/ 1 pts

A cylinder $x^2 + y^2 = 16$ is intersected by the plane $3x + 2y + z = 9$. Find the parameterization of the curve of intersection.

☒ $\langle 4 \cos t, 4 \sin t, 9 - 12 \cos t - 8 \sin t \rangle, t: [0, 2\pi]$

☐ $\langle 4 \cos t, 4 \sin t, 9 - 12 \cos t - 8 \sin t \rangle$

☐ $\langle 4 \cos t, 4 \sin t, 9 + 12 \cos t + 8 \sin t \rangle, t: [0, 2\pi]$

☐ $\langle 4 \cos t, 4 \sin t, 9 + 12 \cos t + 8 \sin t \rangle$

Start with x and y, use $x = 4 \cos t$, $y = 4 \sin t$. Solve for z: $z = 9 - 3x - 2y$, so $z = 9 - 3(4 \cos t) - 2(4 \sin t)$. After simplification, you get $\langle 4 \cos t, 4 \sin t, 9 - 12 \cos t - 8 \sin t \rangle$. Bounds are 0 to 2π .

Part B

Given three points $A = (2, -4, 1)$, $B = (4, 7, 2)$ and $C = (3, 6, 8)$.

- a) Find the equation in parametric form of the line passing through A and B.
- b) Find the shortest distance from C to the line AB. Exact form or decimal approximation (to 2 places) is acceptable.
- c) Find the equation of the plane containing A, B and C. Leave answer in $ax + by + cz = d$ format.

(Pts: 2, 1, 2)

- A) $V = AB = \langle 2, 11, 1 \rangle$, so the line is $L: \langle 2, -4, 1 \rangle + t\langle 2, 11, 1 \rangle$ or $\langle 2 + 2t, -4 + 11t, 1 + t \rangle$. No bounds necessary. Other possible answers, such as using B as the pass through point and any parallel vector for the direction.
- B) Pick point $P = (2, -4, 1)$, form vector $PC = \langle 1, 10, 7 \rangle$. This will be projected onto $v = \langle 2, 11, 1 \rangle$. This is $\frac{119}{126}v$.
This is $v = \langle \frac{238}{126}, \frac{1309}{126}, \frac{119}{126} \rangle$. The opp leg is $PC - v = \langle 1, 10, 7 \rangle - \langle 1.889, 10.389, 0.945 \rangle = \langle -0.889, -0.389, 6.055 \rangle$. The magnitude is 6.132 units.
- C) $AB = \langle 2, 11, 1 \rangle$ and $AC = \langle 1, 10, 7 \rangle$, $AB \times AC = \langle 67, -13, 9 \rangle$. This is the n vector. Using point A, the plane is $67(x - 2) - 13(y + 4) + 9(z - 1) = 0$. After simplification, it is $67x - 13y + 9z = 195$.