

Quiz 5 key

Question 1

1 / 1 pts

What is the equation of the tangent plane to the surface $f(x, y) = 3x^2y^4$ at $x_0 = -2$ and $y_0 = 3$?

(Select the correct letter)

(A) $972(x + 2) + 1296(y - 3) - (z - 972) = 0$

(B) $-972(x + 2) + 1296(y - 3) - (z - 972) = 0$

(C) $972(x - 2) + 1296(y + 3) - (z - 972) = 0$

(D) $-972(z - 2) + 1296(y + 3) - (z - 972) = 0$



B

Normal vector $\mathbf{n} = \langle f_x, f_y, -1 \rangle = \langle 6xy^4, 12x^2y^3, -1 \rangle$. Evaluate at -2 and 3, you get $\mathbf{n} = \langle -972, 1296, -1 \rangle$

Question 2

1 / 1 pts

A pyramid has a square base, and triangular sides that meet at an apex. An archaeologist measures the true vertical height of the pyramid to be 125 meters from base to apex, and the length of each side of the square base to be 150 meters. Suppose the measurements have a tolerance of 0.05 meter for the heights, and 0.04 meter for the length of each side of the base. Use differentials to find the approximate change in volume taking into account the tolerances.

The volume of a square-based pyramid is

$$V(b, h) = \frac{1}{3}(\text{area base})(\text{height}).$$

Enter your answer as an integer.



875

$$V(b, h) = \frac{1}{3}b^2h, \text{ so } dV = V_b db + V_h dh = \frac{2}{3}bh db + \frac{1}{3}b^2 dh. \text{ Thus, } dV = \frac{2}{3}(150)(125)(0.04) + \frac{1}{3}(150)^2(0.05)$$

Question 3

1 / 1 pts

Suppose f is a function of x and y , and x and y are each functions of p , q and r .

Suppose also that $\frac{\partial f}{\partial p} = 30$, $\frac{\partial f}{\partial q} = 24$, $\frac{\partial f}{\partial r} = 15$, $\frac{\partial f}{\partial y} = 5$, $\frac{\partial x}{\partial p} = -2$, $\frac{\partial y}{\partial p} = 4$, $\frac{\partial x}{\partial q} = 9$, $\frac{\partial y}{\partial q} = 3$, $\frac{\partial x}{\partial r} = 10$, $\frac{\partial y}{\partial r} = -8$ and $\frac{\partial y}{\partial r} = 14$. Find $\frac{\partial f}{\partial x}$.

Enter your answer as an integer



-5

The derivative is $\frac{\partial f}{\partial p} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial p} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial p} \rightarrow 30 = \frac{\partial f}{\partial x} (-2) + (5)(4) \rightarrow \frac{\partial f}{\partial x} = -5$.

Because of the repeated partial derivative which could have been confusing, I gave everyone credit for this question as long as it was attempted.

Question 4

1 / 1 pts

Let $f(x, y) = xy^2 + 2x^2y$. What is the direction of steepest ascent, and what is the slope of steepest ascent, at $x_0 = 3$ and $y_0 = 2$?

Select the letter of the correct choice.

- (A) $\langle 28, 30 \rangle, 1684$
- (B) $\langle 33, 30 \rangle, \sqrt{1684}$
- (C) $\langle 28, 30 \rangle, \sqrt{1684}$
- (D) $\langle 33, 30 \rangle, 1684$



C

$$\nabla f = \langle y^2 + 4xy, 2xy + 2x^2 \rangle \rightarrow \nabla f(3, 2) = \langle 28, 30 \rangle, \text{slope} = |\langle 28, 30 \rangle| = \sqrt{(28^2 + 30^2)}$$

Question 5

1 / 1 pts

Let $g(x, y) = x^3 + 2xy^2$. What is the slope of the tangent line at $x_0 = 5, y_0 = 4$, in the direction of the point $(8, 8)$?

Enter your answer as a reduced fraction or a decimal answer to one-decimal place.



128.2

641/5

128.2

$\nabla g = \langle 3x^2 + 2y^2, 4xy \rangle \rightarrow \nabla g(5, 4) = \langle 107, 80 \rangle$. Direction is $\langle 8 - 5, 8 - 4 \rangle = \langle 3, 4 \rangle$, the unit vector is $\langle \frac{3}{5}, \frac{4}{5} \rangle$. Thus the derivative is $\langle 107, 80 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = \frac{321}{5} + \frac{320}{5} = \frac{641}{5}$.

Part B

1. A grain silo on a farm is in the shape of a circular cylinder topped by a hemisphere. A heavy-metal guitarist, driving through on his way to a gig, gets out to measure it. The cylinder part is 8 meters in radius and 30 meters tall. The hemisphere on top would have the same radius, 8 meters. Assume he measured each dimension to within 0.05 meter (the tolerance). Use differentials to determine the change in volume when the tolerances are accounted for. Hint: you'll need to figure out the volume formula of the silo first. (2pts)

2. At a point x_0, y_0 , a function $f(x, y)$ has a slope of $2\sqrt{5}$ in the direction of $\langle 1, 2 \rangle$ and a slope of $\frac{17}{\sqrt{13}}$ in the direction of $\langle 2, 3 \rangle$. Find the direction of steepest ascent, and the slope of steepest ascent, at this point. (3 pts).

- Volume is $V(r, h) = \pi r^2 h + \frac{2}{3} \pi r^3$ (a cylinder + half a sphere). Thus, $dV = V_r dr + V_h dh$, which is $dV = (2\pi r h + 2\pi r^2) dr + (\pi r^2) dh$. Substitute: $dV = (2\pi(8)(30) + 2\pi(8)^2)(0.05) + (\pi(8)^2)(0.05)$. This is $dV = (608\pi)(0.05) + 64\pi(0.05) = 672\pi(0.05) = 33.6\pi \approx 105.56$ cubic meters. Any correct equivalent is acceptable: it can be left in pi form or approximation form.
- The gradient is $\nabla f = \langle f_x, f_y \rangle$. The first direction unit vector is $\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$, so the derivative is $\langle f_x, f_y \rangle \cdot \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle = 2\sqrt{5}$. When dotted out, this is $f_x \left(\frac{1}{\sqrt{5}} \right) + f_y \left(\frac{2}{\sqrt{5}} \right) = 2\sqrt{5}$. The other direction unit vector is $\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \rangle$, and when dotted with the gradient, gives $f_x \left(\frac{2}{\sqrt{13}} \right) + f_y \left(\frac{3}{\sqrt{13}} \right) = \frac{17}{\sqrt{13}}$. Multiply the first by $\sqrt{5}$ and the second by $\sqrt{13}$, and this gives the system
$$\begin{aligned} f_x(1) + f_y(2) &= 10 \\ f_x(2) + f_y(3) &= 17 \end{aligned}$$
 Use any method to solve, you get $f_x = 4, f_y = 3$. Thus, the direction of steepest ascent is $\nabla f = \langle 4, 3 \rangle$ and the slope is $|\nabla f| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.