**Quiz 7 Key**

**Part A**

## Question 1

1 / 1 pts

Region R is bounded by $x^2 + y^2 = 25$ such that $y \geq 0$. What would be its equivalent bounds in polar form?

(Select the correct letter)

(A) $-5 \leq r \leq 5, 0 \leq \theta \leq \pi$

(B) $-5 \leq r \leq 5, 0 \leq \theta \leq 2\pi$

(C) $0 \leq r \leq 5, 0 \leq \theta \leq \pi$

(D) $0 \leq r \leq 5, 0 \leq \theta \leq 2\pi$

C

## Question 2

1 / 1 pts

Evaluate the following exactly:

$$\int_0^2 \int_{\sqrt{4-x^2}}^{\sqrt{36-x^2}} \sqrt{4 + x^2 + y^2}\, dy\, dx + \int_2^6 \int_0^{\sqrt{36-x^2}} \sqrt{4 + x^2 + y^2}\, dy\, dx$$

(A) $\frac{1}{3}\left(40^{1.5} - 8^{1.5}\right)$

(B) $\frac{\pi}{6}\left(40^{1.5} - 8^{1.5}\right)$

(C) $\frac{\pi}{12}\left(40^{1.5} - 8^{1.5}\right)$

(D) $\frac{2\pi}{3}\left(40^{1.5} - 8^{1.5}\right)$

B

The following compound integral, when written in polar form, is...

$$\int_0^{3\sqrt{2}} \int_0^x \left(1 - x^2 - y^2\right) dy\, dx + \int_{3\sqrt{2}}^6 \int_0^{\sqrt{36-x^2}} \left(1 - x^2 - y^2\right) dy\, dx$$

(Select the letter)

(A) $\int_0^{\frac{\pi}{4}} \int_0^6 \left(r - r^3\right) dr d\theta$

(B) $\int_0^{\frac{\pi}{2}} \int_0^6 \left(1 - r^2\right) dr d\theta$

(C) $\int_0^{\frac{\pi}{4}} \int_0^6 \left(1 - r^2\right) dr d\theta$

(D) $\int_0^{\frac{\pi}{2}} \int_0^6 \left(r - r^3\right) dr d\theta$

A

## Question 4

Evaluate this triple integral:

$$\int_{-1}^2 \int_0^4 \int_2^3 x^2 y z^3 \, dz\, dy\, dx$$

Enter your answer as an integer

390

## Question 5

1 / 1 pts

Find the volume inside of the inverted paraboloid $f(x, y) = 4 - x^2 - y^2$ that lies above the xy-plane.

Enter your answer in exact form. Use "pi" for $\pi$

8pi

**Part B**

1. Evaluate $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{1}{1+x^2+y^2}\,dy\,dx$. State your answer in exact form. [3 pts]

The region is a half circle of radius 3 to the right of the y-axis, so $0 \le r \le 3, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. The integral is polar form is $\int_{-\pi/2}^{\pi/2} \int_0^3 \frac{1}{1+r^2}\, r\, dr\, d\theta$. The inner integral is $\int_0^3 \frac{1}{1+r^2}\, r\, dr = \left[\frac{1}{2}\ln(1+r^2)\right]_0^3 = \frac{1}{2}\ln 10$. The outer integral is $\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$, so the correct answer is $\frac{\pi}{2}\ln 10$.

2. Rewrite $\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sqrt{25-x^2-y^2}\,dy\,dx$ in polar form. Do not evaluate. [2 pts]

The region is a full circle centered at the origin, radius 4. The bounds are $0 \le r \le 4, 0 \le \theta \le 2\pi$. The full integral is $\int_0^{2\pi} \int_0^4 \sqrt{25-r^2}\, r\, dr\, d\theta$.