## MAT 267 Quiz 1

## Name:

Take an image of your sheet, make sure it's readable, save as PDF and submit to the assignment site in Canvas by midnight. **Be neat! Show work.** 

1. The sphere  $(x - 1)^2 + (y + 3)^2 + (z - 4)^2 = 58$  intersects the *xz*-plane and forms a circle. Find the radius of this circle. [3 pts]

Set y=0, so we have  $(x - 1)^2 + (0 + 3)^2 + (z - 4)^2 = 58$  which is  $(x - 1)^2 + 9 + (z - 4)^2 = 58$ , or  $(x - 1)^2 + (z - 4)^2 = 49$ . Thus, the radius of the circle is 7.

2. Find the coordinates of (a, b) using projections. [5 pts]



Let  $\mathbf{u} = \text{vector from } (2,3) \text{ to } (3,5)$ , so  $\mathbf{u} = \langle 1,2 \rangle$  and let  $\mathbf{v}$  be vector from (2,3) to (10,8), so  $\mathbf{v} = \langle 8,5 \rangle$ . Note that  $\mathbf{v}$  is the hypotenuse. Always project the hypotenuse onto the adjacent.

Project v onto u:

$$\operatorname{proj}_{\mathbf{u}}\mathbf{v} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \cdot \mathbf{u}}\right)\mathbf{u} = \frac{18}{5}\mathbf{u} = \left\langle\frac{18}{5}, \frac{36}{5}\right\rangle$$

Then offset from the starting point (2,3), so the point (*a*,*b*) is  $\left(2 + \frac{18}{5}, 3 + \frac{36}{5}\right) = \left(\frac{28}{5}, \frac{51}{5}\right) = (5.6, 10.2)$ 

Any correct equivalent is acceptable.

3. For what value of k are the vectors  $\mathbf{u} = \langle 1, 2, k \rangle$  and  $\mathbf{v} = \langle k, -5, 4 \rangle$  acute? Leave answer in interval form or as an inequality. [2 pts]

Dot product:  $\mathbf{u} \cdot \mathbf{v} = (1)(k) + (2)(-5) + (k)(4) = k - 10 + 4k = 5k - 10$ . The dot product of acute vectors will be positive, so  $5k - 10 > 0 \rightarrow 5k > 10$ , so k > 2 or the interval  $(2, \infty)$ .