MAT267 Quiz 3 **KEY** **Name:**

Show all work, be neat.

1. Find the equation of the line in R^3 that is tangent to $\mathbf{r}(t) = \langle t^2 - 3,\ \sqrt{t},\ \frac{1}{t}\rangle$ at $t = 4$. Leave your answer in $\langle x, y, z\rangle + t\langle a, b, c\rangle$ form.

3 pts

Point: $r(4) = \langle 13, 2, \frac{1}{4}\rangle$.

Derivative: $r'(t) = \langle 2t, \frac{1}{2\sqrt{t}}, -\frac{1}{t^2}\rangle$, so $r'(4) = \langle 8, \frac{1}{4}, -\frac{1}{16}\rangle$.

Line is $\langle 13, 2, \frac{1}{4}\rangle + t\,\langle 8, \frac{1}{4}, -\frac{1}{16}\rangle$

2. Given $\mathbf{r}'(t) = \langle\ 2t,\ t^2 + 1, \frac{1}{t}\ \rangle$, find $\int \mathbf{r}'(t)\,dt$ such that $\mathbf{r}(1) = \langle 2,3,4\rangle$.

3 pts

$$\int \langle\ 2t,\ t^2 + 1, \frac{1}{t}\rangle\ dt = \langle t^2, \frac{1}{3}t^3 + t, \ln t\rangle + \langle a, b, c\rangle$$

$$r(1):\quad \langle 1, \frac{4}{3}, 0\rangle + \langle a, b, c\rangle = \langle 2,3,4\rangle \rightarrow \begin{array}{l} 1 + a = 2 \rightarrow a = 1 \\ \frac{4}{3} + b = 3 \rightarrow b = \frac{5}{3} \\ 0 + c = 4 \rightarrow c = 4 \end{array}$$

Answer: $\langle t^2, \frac{1}{3}t^3 + t, \ln t\rangle + \langle 1, \frac{5}{3}, 4\rangle$

3. Let $\mathbf{r}(t) = \langle\ t,\ t^2, t^3\ \rangle$.

   a) Find the arc length for $0 \le t \le 5$. For full credit, show the entire integral set-up.

2 pts

$$\int_0^5 |r'(t)|dt = \int_0^5 \sqrt{1^2 + (2t)^2 + (3t^2)^2}\ dt = \int_0^5 \sqrt{1 + 4t^2 + 9t^4}\ dt\ \approx 128.598$$

   b) Find $\mathbf{T}(1)$.

2 pts

$$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{\langle 1, 2t, 3t^2\rangle}{\sqrt{1 + 4t^2 + 9t^4}} \rightarrow @t = 1 \rightarrow \langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$$